THE GALOIS ALGEBRAS AND THE AZUMAYA GALOIS EXTENSIONS

GEORGE SZETO and **LIANYONG XUE**

Department of Mathematics, Bradley University
Peoria, Illinois 61625 – U.S.A.
Email: szeto@hilltop.bradley.edu and lxue@hilltop.bradley.edu

ABSTRACT. Let B be a Galois algebra over a commutative ring R with Galois group G, C the center of B, $K = \{g \in G \,|\, g(c) = c \text{ for all } c \in C\}$, $J_g = \{b \in B \,|\, bx = g(x)b \text{ for all } x \in B\}$ for each $g \in K$, and $B_K = \oplus \sum_{g\in K} J_g$. Then B_K is a central weakly Galois algebra with Galois group induced by K. Moreover, an Azumaya Galois extension B with Galois group K is characterized by using B_K.

Keywords and phrases. Galois algebras, central Galois algebras, weakly Galois extensions, Azumaya Galois extensions, and Azumaya weakly Galois extensions.

2000 Mathematics Subject Classification. Primary 16S35, 16W20.

1. Introduction. Let B be a Galois algebra over a commutative ring R with Galois group G and C the center of B. The class of Galois algebras has been investigated by F. R. DeMeyer ([2]), T. Kanzaki ([6]), M. Harada ([4], [5]), and the authors ([7]). In [2], it was shown that if R contains no idempotents but 0 and 1, then B is a central Galois algebra with Galois group K and C is a commutative Galois algebra with Galois group G/K where $K = \{g \in G \,|\, g(c) = c \text{ for all } c \in C\}$ ([2], Theorem 1). This fact was extended to the Galois algebra B over R containing more than two idempotents ([6], Proposition 3), and generalized to any Galois algebra B ([7], Theorem 3.8) by using the Boolean algebra B_a generated by $\{0, e_g \,|\, g \in G$ for a central idempotent $e_g\}$ where $BJ_g = Be_g$ and $J_g = \{b \in B \,|\, bx = g(x)b \text{ for all } x \in B\}$ for each $g \in G$ ([6]). The purpose of the present paper is to show that there exists a subalgebra B_K of B such that B_K is a central weakly Galois algebra with Galois group $K|_{B_K}$ induced by K where a weakly Galois algebra was defined in ([8]) and that $B_K B^K$ is an Azumaya weakly Galois extension with Galois group $K|_{B_K B^K}$ where an Azumaya Galois extension was studied in ([1]). Thus some characterizations of an Azumaya Galois extension B of B^K with Galois group K are obtained, and the results as given in [2] and [6] are generalized.

2. Definitions and notations. Throughout, let B be a Galois algebra over a commutative ring R with Galois group G, C the center of B, and $K = \{g \in G \,|\, g(c) = c$ for all $c \in C\}$. We keep the definitions of a Galois extension, a Galois algebra, a central Galois algebra, a separable extension, and an Azumaya algebra as defined in ([7]). An Azumaya Galois extension A with Galois group G is a Galois extension A of A^G which is a C^G-Azumaya algebra where C the center of A ([1]). A weakly Galois extension A with Galois group G is a finitely generated projective left module A over A^G such that $A_l G \cong \mathrm{Hom}_{A^G}(A, A)$ where $A_l = \{a_l$, a left multiplication map by $a \in A\}$ ([8]). We call that A is a weakly Galois algebra with Galois group G if A is a weakly Galois extension with Galois group G such that A^G is contained in the center of A and that A is a central weakly Galois algebra with Galois group G if A is a weakly Galois extension with Galois group G such that A^G is the center of A. An Azumaya weakly Galois extension A with Galois group G is a weakly Galois extension A of A^G which is a C^G-Azumaya algebra where C the center of A.

3. A weakly Galois algebra. In this section, let B be a Galois algebra over R with Galois group G, C the center of B, $B^G = \{b \in B \,|\, g(b) = b$ for all $g \in G\}$, and $K = \{g \in G \,|\, g(c) = c$ for all $c \in C\}$. Then, $B = \oplus \sum_{g \in G} J_g = (\oplus \sum_{g \in K} J_g) \oplus (\oplus \sum_{g \notin K} J_g)$ where $J_g = \{b \in B \,|\, bx = g(x)b$ for all $x \in B\}$ ([6], Theorem 1). We denote $\oplus \sum_{g \in K} J_g$ by B_K and the center of B_K by Z. Clearly, K is a normal subgroup of G. We shall show that

B_K is an Azumaya algebra over Z and a central weakly Galois algebra with Galois group $K|_{B_K}$.

THEOREM 3.1. *B_K is an Azumaya algebra over Z.*

PROOF. By the definition of B_K, $B_K = \oplus \sum_{g\in K} J_g$, so $C(= J_1) \subset B_K$. Since B is a Galois algebra with Galois group G and $K = \{g \in G \,|\, g(c) = c \text{ for all } c \in C\}$, the order of K is a unit in C by Proposition 5 in [6]. Moreover, K is an C-automorphism group of B, so B_K is a C-separable algebra by Proposition 5 in [5]. Thus B_K is an Azumaya algebra over Z.

In order to show that B_K is a central weakly Galois algebra with Galois group $K|_{B_K}$, we need two lemmas.

LEMMA 3.2. *Let $L = \{g \in K \,|\, g(a) = a \text{ for all } a \in B_K\}$. Then, L is a normal subgroup of K such that $\overline{K}(= K/L)$ is an automorphism group of B_K induced by K (that is, $K|_{B_K} \cong \overline{K}$).*

PROOF. Clearly, L is a normal subgroup of K, so for any $h \in K$, $h(B_K) = \oplus \sum_{g\in K} h(J_g) = \oplus \sum_{g\in K} J_{hgh^{-1}} = \oplus \sum_{g\in hKh^{-1}} J_g = \oplus \sum_{g\in K} J_g = B_K$. Thus $K|_{B_K} \cong \overline{K}$.

LEMMA 3.3. $(B_K)^K = Z$.

PROOF. Let x be any element in $(B_K)^K$ and b any element in B_K. Then $b = \sum_{g\in K} b_g$ where $b_g \in J_g$ for each $g \in K$. Hence $bx = \sum_{g\in K} b_g x = \sum_{g\in K} g(x) b_g = \sum_{g\in K} x b_g = x \sum_{g\in K} b_g = xb$. Therefore $x \in Z$. Thus $(B_K)^K \subset Z$. Conversely, for any $z \in Z$ and $g \in K$, we have that $zx = xz = g(z)x$ for any $x \in J_g$, so $(g(z) - z)x = 0$ for any $x \in J_g$. Hence $(g(z) - z)J_g = \{0\}$. Noting that $BJ_g = J_g B = B$, we have that

$(g(z)-z)B=\{0\}$, so $g(z)=z$ for any $z\in Z$ and $g\in K$. Thus $Z\subset (B_K)^K$. Therefore $(B_K)^K=Z$.

THEOREM 3.4. *B_K is a central weakly Galois algebra with Galois group $K|_{B_K}\cong \overline{K}$.*

PROOF. By Lemma 3.3, it suffices to show that (1) B_K is a finitely generated projective module over Z, and (2) $(B_K)_l\overline{K}\cong \mathrm{Hom}_Z(B_K,B_K)$. Part (1) is a consequence of Theorem 3.1. For part (2), since B_K is an Azumaya algebra over Z by Theorem 3.1 again, $B_K\otimes_Z B_K^o\cong \mathrm{Hom}_Z(B_K,B_K)$ ([3], Theorem 3.4, page 52) by extending the map $(a\otimes b)(x)=axb$ linearly for $a\otimes b\in B_K\otimes_Z B_K^o$ and each $x\in B_K$ where B_K^o is the opposite algebra of B_K. By denoting the left multiplication map with $a\in B_K$ by a_l and the right multiplication map with $b\in B_K$ by b_r, $(a\otimes b)(x)=(a_lb_r)(x)=axb$. Since $B_K=\oplus\sum_{g\in K}J_g$, $B_K\otimes_Z B_K^o=\sum_{g\in K}(B_K)_l(J_g)_r$. Observing that $(J_g)_r=(J_g)_l\bar{g}^{\,-1}$ where $\overline{g}=g|_{B_K}\in K|_{B_K}\cong\overline{K}$, we have that $B_K\otimes_Z B_K^o=\sum_{g\in K}(B_K)_l(J_g)_r=\sum_{g\in K}(B_K)_l(J_g)_l\bar{g}^{\,-1}=\sum_{g\in K}(B_KJ_g)_l\bar{g}^{\,-1}$. Moreover, since $BJ_g=B$ for each $g\in K$ and $B=\oplus\sum_{h\in G}J_h=B_K\oplus(\oplus\sum_{h\notin K}J_h)$, $B_K\oplus(\oplus\sum_{h\notin K}J_h)=B=BJ_g=B_KJ_g\oplus(\oplus\sum_{h\notin K}J_hJ_g)$ such that $B_KJ_g\subset B_K$ and $\oplus\sum_{h\notin K}J_hJ_g\subset\oplus\sum_{h\notin K}J_h$. Hence $B_KJ_g=B_K$ for each $g\in K$. Therefore $B_K\otimes_Z B_K^o=\sum_{g\in K}(B_KJ_g)_l\bar{g}^{\,-1}=\sum_{g\in K}(B_K)_l\bar{g}^{\,-1}=(B_K)_l\overline{K}$. Thus $(B_K)_l\overline{K}\cong\mathrm{Hom}_Z(B_K,B_K)$. This completes the proof of part (2). Thus B_K is a central weakly Galois algebra with Galois group $K|_{B_K}\cong\overline{K}$.

Recall that an algebra A is called an Azumaya weakly Galois extension of A^K with Galois group K if A is a weakly Galois extension of A^K which is a C^K-Azumaya algebra where C is the center of A. Next, we show that B_KB^K is an Azumaya weakly Galois extension with Galois group $K|_{B_KB^K}\cong\overline{K}$. We begin with two lemmas about B_K.

LEMMA 3.5. $B^K=V_B(B_K)$.

PROOF. For any $b \in B^K$ and $x \in J_g$ for any $g \in K$, we have that $xb = g(b)x = bx$, so $b \in V_B(J_g)$ for any $g \in K$. Thus $b \in V_B(B_K)$. Conversely, for any $b \in V_B(B_K)$ and $g \in K$, we have that $bx = xb = g(b)x$ for any $x \in J_g$, so $(g(b) - b)x = 0$ for any $x \in J_g$. Hence $(g(b) - b)J_g = \{0\}$. But $BJ_g = J_gB = B$ for any $g \in K$, so $(g(b) - b)B = \{0\}$. Thus $g(b) = b$ for any $g \in K$; and so $b \in B^K$. Therefore $B^K = V_B(B_K)$.

LEMMA 3.6. *B^K is an Azumaya algebra over Z where Z is the center of B_K.*

PROOF. Since B is a Galois algebra over R with Galois group G, B is an Azumaya algebra over its center C. By the proof of Theorem 3.1, B_K is a C-separable subalgebra of B, so $V_B(B_K)$ is a C-separable subalgebra of B and $V_B(V_B(B_K)) = B^K$ by the commutator theorem for Azumaya algebras ([3], Theorem 4.3, page 57). This implies that B_K and $V_B(B_K)$ have the same center Z. Thus $V_B(B_K)$ is an Azumaya algebra over Z. But, by Lemma 3.5, $B^K = V_B(B_K)$, so B^K is an Azumaya algebra over Z.

THEOREM 3.7. *Let $A = B_K B^K$. Then A is an Azumaya weakly Galois extension with Galois group $K|_A \cong \overline{K}$.*

PROOF. Since B_K is a central weakly Galois algebra with Galois group $K|_{B_K} \cong \overline{K}$ by Theorem 3.4, B_K is a finitely generated projective module over Z and $(B_K)_l\overline{K} \cong \mathrm{Hom}_Z(B_K, B_K)$. By Lemma 3.6, B^K is an Azumaya algebra over Z, so $A(\cong B_K \otimes_Z B^K)$ is a finitely generated projective module over $B^K(= A^{\overline{K}})$. Moreover, since $B^K = V_B(B_K)$ by Lemma 3.5 and $(B_K)_l\overline{K} \cong \mathrm{Hom}_Z(B_K, B_K)$,

$$\begin{aligned} A_l\overline{K} &= (B_K B^K)_l\overline{K} = (B_K)_l\overline{K}(B^K)_r \cong B_K\overline{K} \otimes_Z B^K \cong \mathrm{Hom}_Z(B_K, B_K) \otimes_Z B^K \\ &\cong \mathrm{Hom}_{B^K}(B_K \otimes_Z B^K, B_K \otimes_Z B^K) \cong \mathrm{Hom}_{B^K}(B_K B^K, B_K B^K) \\ &= \mathrm{Hom}_{A^{\overline{K}}}(A, A). \end{aligned}$$

Thus A is a weakly Galois extension of A^K with Galois group $K|_A \cong \overline{K}$. Next we claim that A has center Z and $A^{\overline{K}}$ is an Azumaya algebra over $Z^{\overline{K}}$. In fact, B_K and B^K are

Azumaya algebras over Z by Theoerm 3.1 and Lemma 3.6 respectively, so $A(= B_K B^K)$ has center Z and $A^{\overline{K}} = (B_K B^K)^{\overline{K}} = B^K$. Noting that B^K is an Azumaya algebra over Z, we conclude that $A^{\overline{K}}$ is an Azumaya algebra over $Z^{\overline{K}}$. Thus A is an Azumaya weakly Galois extension with Galois group $K|_A \cong \overline{K}$.

4. An Azumaya Galois extension.

In this section, we shall give several characterizations of an Azumaya Galois extension B by using B_K. This generalizes the results as given in [2] and [6]. The Z-module $\{b \in B_K \,|\, bx = g(x)b \text{ for all } x \in B_K\}$ is denoted by $J_{\overline{g}}^{(B_K)}$ for $\overline{g} \in \overline{K}$ where $\overline{K}(= K/L)$ is defined in Lemma 3.2.

LEMMA 4.1. *B_K is a central Galois algebra with Galois group $K|_{B_K} \cong \overline{K}$ if and only if $J_{\overline{g}}^{(B_K)} = \oplus \sum_{l \in L} J_{gl}$ for each $\overline{g} \in \overline{K}$.*

PROOF. Let B_K be a central Galois algebra with Galois group $K|_{B_K} \cong \overline{K}$. Then $B_K = \oplus \sum_{\overline{g} \in \overline{K}} J_{\overline{g}}^{(B_K)}$ ([6], Theorem 1). Next it is easy to check that $\oplus \sum_{l \in L} J_{gl} \subset J_{\overline{g}}^{(B_K)}$. But $B_K = \oplus \sum_{g \in K} J_g$, so $\oplus \sum_{g \in K} J_g = \oplus \sum_{\overline{g} \in \overline{K}} J_{\overline{g}}^{(B_K)}$ where $\oplus \sum_{l \in L} J_{gl} \subset J_{\overline{g}}^{(B_K)}$. Thus $J_{\overline{g}}^{(B_K)} = \oplus \sum_{l \in L} J_{gl}$ for each $\overline{g} \in \overline{K}$. Conversely, since $J_{\overline{g}}^{(B_K)} = \oplus \sum_{l \in L} J_{gl}$ for each $\overline{g} \in \overline{K}$, $B_K = \oplus \sum_{g \in K} J_g = \oplus \sum_{\overline{g} \in \overline{K}} J_{\overline{g}}^{(B_K)}$. Moreover, by Lemma 3.3, $(B_K)^K = Z$, so $\overline{K}$ is an Z-automorphism group of B_K. Hence $J_{\overline{g}}^{(B_K)} J_{\overline{g}^{-1}}^{(B_K)} = Z$ for each $\overline{g} \in \overline{K}$. Thus B_K is a central Galois algebra with Galois group $K|_{B_K} \cong \overline{K}$ because B_K is an Azumaya Z-algebra by Theorem 3.1 ([4], Theorem 1).

Next we characterize an Azumaya Galois extension B with Galois group K.

THEOREM 4.2. *The following statements are equivalent:*

(1) B is an Azumaya Galois extension with Galois group K.

(2) $Z = C$.

(3) $B = B_K B^K$.

(4) B_K *is a central Galois algebra over* C *with Galois group* $K|_{B_K} \cong K$.

PROOF. (1) $\Longrightarrow$ (2) Since B is an Azumaya Galois extension with Galois group K, B^K is an C^K-Azumaya algebra. But, by Lemma 3.6, B^K is an Azumaya algebra over Z, so $Z = C^K$. Hence $C \subset Z = C^K \subset C$. Thus $Z = C$.

(2) $\Longrightarrow$ (3) Suppose that $Z = C$. Then, by Theorem 3.1, B_K is an Azumaya algebra over C. Hence by the commutator theorem for Azumaya algebras, $B = B_K V_B(B_K)$ ([3], Theorem 4.3, page 57). But, by Lemma 3.6, $B^K = V_B(B_K)$, so $B = B_K B^K$.

(3) $\Longrightarrow$ (4) By hypothesis, $B = B_K B^K$, so $L = \{1\}$ where L is given in Lemma 3.2. By the proofs of Theorem 3.1 and Lemma 3.6, B_K and B^K are C-separable subalgebras of the Azumaya C-algebra B such that $B = B_K B^K$, so B_K and B^K are Azumaya algebras over C ([3], Theorem 4.4, page 58). Thus C is the center of B_K. Next we claim that $J_g = J_g^{(B_K)}$ for each $g \in K$. In fact, it is clear that $J_g \subset J_g^{(B_K)}$. Conversely, for each $a \in J_g^{(B_K)}$ and $x \in B$ such that $x = yz$ for some $y \in B_K$ and $z \in B^K$, noting that $B^K = V_B(B_K)$, we have that $ax = ayz = g(y)az = g(y)za = g(yz)a = g(x)a$. Thus $J_g^{(B_K)} \subset J_g$. This proves that $J_g = J_g^{(B_K)} (= J_{\overline{g}}^{(B_K)}$ since $L = \{1\}$) for each $g \in K$. Hence B_K is a central Galois algebra over C with Galois group $K|_{B_K} \cong K$ by Lemma 4.1.

(4) $\Longrightarrow$ (1) Since B is a Galois algebra with Galois group G, B is a Galois extension with Galois group K. By hypothesis, B_K is a central Galois algebra over C with Galois group $K|_{B_K} \cong K$, so the center of B_K is C, that is, $Z = C$. Hence B^K is an Azumaya algebra over $C (= C^K)$ by Lemma 3.6. Thus B is an Azumaya Galois extension with Galois group K.

Theorem 4.2 generalizes the following result of T. Kanzaki ([6], Proposition 3)

COROLLARY 4.3. *If* $J_g = \{0\}$ *for each* $g \notin K$, *then* B *is a central Galois algebra* *with Galois group K and C is a Galois algebra with Galois group G/K.*

PROOF. This is the case in Theorem 4.2 that $B = B_K B^K = B_K$ where $B^K = C$.

We conclude the present paper with two examples, one to illustrate the result in Theorem 4.2, and another to show that $Z \neq C$.

EXAMPLE 4.4. Let $A = R[i,j,k]$, the real quaternion algebra over the field of real numbers R, $B = (A \otimes_R A) \oplus A \oplus A \oplus A \oplus A$, and G the group generated by the elements in $\{g_1,\ k_i,\ k_j,\ k_k,\ h_i,\ h_j,\ h_k\}$ where g_1 is the identity of G and for all $(a \otimes b, a_1, a_2, a_3, a_4) \in B$,

$$k_i(a \otimes b, a_1, a_2, a_3, a_4) = (iai^{-1} \otimes b\,, ia_1i^{-1}, ia_2i^{-1}, ia_3i^{-1}, ia_4i^{-1}),$$
$$k_j(a \otimes b, a_1, a_2, a_3, a_4) = (jaj^{-1} \otimes b\,, ja_1j^{-1}, ja_2j^{-1}, ja_3j^{-1}, ja_4j^{-1}),$$
$$k_k(a \otimes b, a_1, a_2, a_3, a_4) = (kak^{-1} \otimes b\,, ka_1k^{-1}, ka_2k^{-1}, ka_3k^{-1}, ka_4k^{-1}),$$
$$h_i(a \otimes b, a_1, a_2, a_3, a_4) = (a \otimes ibi^{-1}\,, a_2, a_1, a_4, a_3),$$
$$h_j(a \otimes b, a_1, a_2, a_3, a_4) = (a \otimes jbj^{-1}\,, a_3, a_4, a_1, a_2),$$
$$h_k(a \otimes b, a_1, a_2, a_3, a_4) = (a \otimes kbk^{-1}\,, a_4, a_3, a_2, a_1).$$

Then,

(1) We can check that B is a Galois algebra over B^G with Galois group G where $B^G = \{(r_1 \otimes r_2, r, r, r, r) \,|\, r_1, r_2, r \in R\} \subset C$, and $C = (R \otimes R) \oplus R \oplus R \oplus R \oplus R$, the center of B.

(2) $K = \{g \in G \,|\, g(c) = c \text{ for all } c \in C\} = \{g_1,\ k_i,\ k_j,\ k_k\}$.

(3) $J_1 = C$, $J_{k_i} = (Ri \otimes 1) \oplus Ri \oplus Ri \oplus Ri \oplus Ri$, $J_{k_j} = (Rj \otimes 1) \oplus Rj \oplus Rj \oplus Ri \oplus Rj$, $J_{k_k} = (Rk \otimes 1) \oplus Rk \oplus Rk \oplus Ri \oplus Rk$, so $B_K = (A \otimes_R R) \oplus A \oplus A \oplus A \oplus A$. Hence B_K has center C, that is $Z = C$, and B_K is a central Galois algebra over C with Galois group $K|_{B_K} \cong K$.

(4) $B^K = (R \otimes A) \oplus R \oplus R \oplus R \oplus R)$ and $B = B_K B^K$, that is, B is an Azumaya Galois extension with Galois group K.

EXAMPLE 4.5. Let $A = R[i,j,k]$, the real quaternion algebra over the field of real numbers R, $B = A \oplus A \oplus A$, $G = \{1,\ g_i,\ g_j,\ g_k,\}$, and for all $(a_1,a_2,a_3) \in B$,

$g_i(a_1,a_2,a_3) = (ia_1i^{-1}, ia_2i^{-1}, ia_3i^{-1})$,

$g_j(a_1,a_2,a_3) = (ja_1j^{-1}, ja_3j^{-1}, ja_2j^{-1})$,

$g_k(a_1,a_2,a_3) = (ka_1k^{-1}, ka_3k^{-1}, ka_2k^{-1})$,

Then,

(1) B is a Galois algebra over B^G where $B^G = \{(r_1,r,r)\,|\,r_1,r \in R\} \subset C$, and $C = R \oplus R \oplus R$, the center of B. The G-Galois system is $\{a_i;\, b_i\,|\,i = 1,2,\cdots,8\}$ where $a_1 = (1,0,0)$, $a_2 = (i,0,0)$, $a_3 = (j,0,0)$, $a_4 = (k,0,0)$, $a_5 = (0,1,0)$, $a_6 = (0,j,0)$, $a_7 = (0,0,1)$, $a_8 = (0,0,k)$; $b_1 = \frac{1}{4}a_1$, $b_2 = -\frac{1}{4}a_2$, $b_3 = -\frac{1}{4}a_3$, $b_4 = -\frac{1}{4}a_4$, $b_5 = \frac{1}{2}a_5$, $b_6 = -\frac{1}{2}a_6$, $b_7 = \frac{1}{2}a_7$, $b_8 = -\frac{1}{2}a_8$.

(2) $K = \{g \in G\,|\,g(c) = c$ for all $c \in C\} = \{1,\ g_i\}$ where $J_{g_i} = Ri \oplus Ri \oplus Ri$, so $B_K = R[i] \oplus R[i] \oplus R[i]$ which is a commutative ring not equal to C, that is, $Z \neq C$.

ACKNOWLEDGEMENT. This paper was written under the support of a Caterpillar Fellowship at Bradley University. The authors would like to thank the Caterpillar Inc. for the support.

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