

THE GENERAL IKEHATA THEOREM FOR H-SEPARABLE CROSSED PRODUCTS

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Abstract. Let B be a ring with 1, C the center of B , G an automorphism group of B of order n for some integer n , C^G the set of elements in C fixed under G , $\Delta = \Delta(B, G, f)$ a crossed product over B where f is a factor set from $G \times G$ to $U(C^G)$. It is shown that Δ is a H -separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$.

Key Words and Phrases: Crossed products, Galois extensions, H -separable extensions.

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1. INTRODUCTION

Let B be a ring with 1, ρ an automorphism of B of order n , $B[x; \rho]$ a skew polynomial ring with a basis $\{1, x, x^2, \dots, x^{n-1}\}$ and $x^n = v \in U(B^\rho)$ for some integer n , where B^ρ is the set of elements in B fixed under ρ and $U(B^\rho)$ is the set of units of B^ρ .

In [3] it was shown that any skew polynomial ring $B[x; \rho]$ of prime degree n is a H -separable extension of B if and only if C is a Galois algebra over C^ρ with Galois group $\langle \rho|_C \rangle$ generated by $\rho|_C$ of order n . This theorem was extended to any degree n ([5], Theorem

1). Recently, the theorem was completely generalized by the present authors in [8], that is, let $B[x; \rho]$ be a skew polynomial ring of degree n for some integer n . Then, $B[x; \rho]$ is a H -separable extension of B if and only if C is a Galois algebra over C^ρ with Galois group $\langle \rho|_C \rangle \cong \langle \rho \rangle$. The purpose of the present paper is to generalize the above Ikehata theorem to a crossed product $\Delta = \Delta(B, G, f)$ where G is a finite automorphism group of B (not necessarily cyclic) and f is a factor set from $G \times G$ to $U(C^G)$. We shall show that Δ is a H -separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$. This paper was written under the support of a Caterpillar Fellowship at Bradley University. We would like to thank Caterpillar Inc. for the support.

2. PRELIMINARIES AND BASIC DEFINITIONS

Throughout this paper, B will represent a ring with 1, C the center of B , G an automorphism group of B of order n for some integer n , B^G the set of elements in B fixed under G , $\Delta = \Delta(B, G, f)$ a crossed product with a free basis $\{U_g \mid g \in G \text{ and } U_1 = 1\}$ over B and the multiplications are given by $U_g b = g(b)U_g$ and $U_g U_h = f(g, h)U_{gh}$ for $b \in B$ and $g, h \in G$ where f is a map from $G \times G$ to $U(C^G)$ such that $f(g, h)f(gh, k) = f(h, k)f(g, hk)$, Z the center of Δ , \bar{G} the inner automorphism group of Δ induced by G , that is, $\bar{g}(x) = U_g x U_g^{-1}$ for each $x \in \Delta$ and $g \in G$. We note that $f(g, 1) = f(1, g) = f(1, 1) = 1$ for all $g \in G$ and \bar{G} restricted to B is G .

Let A be a subring of a ring S with the same identity 1. We denote $V_S(A)$ the commutator subring of A in S . A ring S is called a G -Galois extension of S^G if there exist elements $\{a_i, b_i \text{ in } S, i = 1, 2, \dots, m\}$ for some integer m such that $\sum_{i=1}^m a_i g(b_i) = \delta_{1, g}$. The set $\{a_i, b_i\}$ is called a G -Galois system for S . S is called a H -separable extension of A if there exists a H -separable system $\{x_i \in V_S(A), y_i \in V_{S \otimes_A S}(S) \mid i = 1, 2, \dots, m\}$ for S over A for some integer m such that $\sum_{i=1}^m x_i y_i = 1 \otimes_A 1$.

3. THE IKEHATA THEOREM

In this section, we shall show that Δ is a H -separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$. We begin with a Lemma.

LEMMA 3.1.

- (a) $V_\Delta(B) = \sum_{g \in G} J_g U_g$ where $J_g = \{b \in B \mid ab = bg(a) \text{ for all } a \in B\}$.
- (b) $V_{\Delta \otimes_B \Delta}(\Delta) = \{\sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h \mid b_{(g,h)} \in J_{gh} \text{ and } k(b_{(k^{-1}g,h)})f(k, k^{-1}g) = b_{(g,hk^{-1})}f(hk^{-1}, k) \text{ for all } g, h, k \in G\}$.
- (c) If $\sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h \in V_{\Delta \otimes_B \Delta}(\Delta)$, then $b_{(g,h)} U_{gh} \in V_\Delta(B)$.
- (d) If $\sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h \in V_{\Delta \otimes_B \Delta}(\Delta)$, then $b_{(g,g^{-1})} = g(b_{(1,1)}(f(g^{-1}, g)))^{-1}$ for all $g \in G$.

PROOF. (a) Let $b \in J_g$. Then $a(bU_g) = (ab)U_g = bg(a)U_g = (bU_g)a$ for all $a \in B$. Hence $J_g U_g \subset V_\Delta(B)$. Therefore, $\sum_{g \in G} J_g U_g \subset V_\Delta(B)$. Conversely, let $\sum_{g \in G} b_g U_g \in V_\Delta(B)$. Then $a \sum_{g \in G} b_g U_g = \sum_{g \in G} b_g U_g a = \sum_{g \in G} b_g g(a) U_g$ for all $a \in B$, and so $ab_g = b_g g(a)$ for all $a \in B$ and $g \in G$, that is, $b_g \in J_g$ for all $g \in G$. Thus $V_\Delta(B) \subset \sum_{g \in G} J_g U_g$.

(b) $x = \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h \in V_{\Delta \otimes_B \Delta}(\Delta)$ if and only if $bx = xb$ and $U_k x = x U_k$ for all $a \in B$ and $k \in G$. But

$$\begin{aligned} bx &= \sum_{g \in G} \sum_{h \in G} b b_{(g,h)} U_g \otimes_B U_h \text{ and} \\ xb &= \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h b = \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B h(b) U_h \\ &= \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g h(b) \otimes_B U_h = \sum_{g \in G} \sum_{h \in G} b_{(g,h)} (gh)(b) U_g \otimes_B U_h, \end{aligned}$$

so $bx = xb$ if and only if $bb_{(g,h)} = b_{(g,h)}((gh)(b))$ for all $b \in B$ and $g, h \in G$, that is,

$b_{(g,h)} \in J_{gh}$ by noting that $\{U_g \otimes_B U_h \mid g, h \in G\}$ is a basis for Δ over B . Moreover,

$$\begin{aligned} U_k x &= U_k \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h = \sum_{g \in G} \sum_{h \in G} k(b_{(g,h)}) U_k U_g \otimes_B U_h \\ &= \sum_{g \in G} \sum_{h \in G} k(b_{(g,h)}) f(k, g) U_{kg} \otimes_B U_h \\ &= \sum_{g \in G} \sum_{h \in G} k(b_{(k^{-1}(kg), h)}) f(k, k^{-1}(kg)) U_{(kg)} \otimes_B U_h \\ &= \sum_{l \in G} \sum_{h \in G} k(b_{(k^{-1}l, h)}) f(k, k^{-1}l) U_l \otimes_B U_h = \sum_{g \in G} \sum_{h \in G} k(b_{(k^{-1}g, h)}) f(k, k^{-1}g) U_g \otimes_B U_h, \end{aligned}$$

and

$$\begin{aligned} x U_k &= \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B U_h U_k = \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes_B f(h, k) U_{hk} \\ &= \sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g f(h, k) \otimes_B U_{hk} = \sum_{g \in G} \sum_{h \in G} b_{(g,h)} f(h, k) U_g \otimes_B U_{hk} \\ &= \sum_{g \in G} \sum_{h \in G} b_{(g, (hk)k^{-1})} f((hk)k^{-1}, k) U_g \otimes_B U_{hk} = \sum_{g \in G} \sum_{l \in G} b_{(g, lk^{-1})} f(lk^{-1}, k) U_g \otimes_B U_l \\ &= \sum_{g \in G} \sum_{h \in G} b_{(g, hk^{-1})} f(hk^{-1}, k) U_g \otimes_B U_h. \end{aligned}$$

Hence, $U_k x = x U_k$ if and only if $k(b_{(k^{-1}g, h)}) f(k, k^{-1}g) = b_{(g, hk^{-1})} f(hk^{-1}, k)$ for all $g, h, k \in G$.

(c) If $\sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes U_h \in V_{\Delta \otimes_B \Delta}(\Delta)$, then $b_{(g,h)} \in J_{gh}$ by (b); and so $b_{(g,h)} U_{gh} \in V_{\Delta}(B)$ by (a).

(d) If $\sum_{g \in G} \sum_{h \in G} b_{(g,h)} U_g \otimes U_h \in V_{\Delta \otimes_B \Delta}(\Delta)$, then $k(b_{(k^{-1}g, h)}) f(k, k^{-1}g) = b_{(g, hk^{-1})} f(hk^{-1}, k)$ for all $g, h, k \in G$ by (b). Let $k = g$ and $h = 1$. Then $b_{(g, g^{-1})} f(g^{-1}, g) = g(b_{(1,1)}) f(g, 1) = g(b_{(1,1)})$ for all $g \in G$. This implies that $b_{(g, g^{-1})} = g(b_{(1,1)}) (f(g^{-1}, g))^{-1}$ for all $g \in G$.

THEOREM 3.2. Δ is a H -separable extension of B and $V_{\Delta}(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$.

PROOF. (\implies) Since Δ is a H -separable extension of B and B is a direct summand of Δ as a left B -module, $V_{\Delta}(V_{\Delta}(B)) = B$ ([7], Proposition 1.2). But $V_{\Delta}(B)$ is commutative, so $V_{\Delta}(B) \subset V_{\Delta}(V_{\Delta}(B)) = B$. Thus $V_{\Delta}(B) = C$.

Since Δ is a H -separable extension of B again, there exists a H -separable system $\{x_i \in V_\Delta(B), y_i \in V_{\Delta \otimes_B \Delta}(\Delta) \mid i = 1, 2, \dots, m\}$ for some integer m such that $\sum_{i=1}^m x_i y_i = 1 \otimes_B 1$. Let $y_i = \sum_{g \in G} \sum_{h \in G} b_{(g,h)}^{(i)} U_g \otimes_B U_h$. We claim that $\{a_i = x_i, b_i = b_{(1,1)}^{(i)} \mid i = 1, 2, \dots, m\}$ is a G -Galois system for C . In fact, $a_i = x_i \in V_\Delta(B) = C$ and by Lemma 3.1-(b), $b_i = b_{(1,1)}^{(i)} \in J_1 = C$. Moreover, since $y_i = \sum_{g \in G} \sum_{h \in G} b_{(g,h)}^{(i)} U_g \otimes_B U_h \in V_{\Delta \otimes_B \Delta}(\Delta)$, $b_{(g,h)}^{(i)} U_{gh} \in V_\Delta(B)$ by Lemma 3.1-(c). But $V_\Delta(B) = C$, so $b_{(g,h)}^{(i)} = 0$ when $gh \neq 1$. Thus, $y_i = \sum_{g \in G} b_{(g,g^{-1})}^{(i)} U_g \otimes_B U_{g^{-1}}$. By Lemma 3.1-(d), $b_{(g,g^{-1})}^{(i)} = g(b_{(1,1)}^{(i)})(f(g^{-1}, g))^{-1} = g(b_i)(f(g^{-1}, g))^{-1}$, so $y_i = \sum_{g \in G} g(b_i)(f(g^{-1}, g))^{-1} U_g \otimes_B U_{g^{-1}}$. Therefore,

$$\begin{aligned} 1 \otimes_B 1 &= \sum_{i=1}^m x_i y_i = \sum_{i=1}^m a_i \sum_{g \in G} g(b_i)(f(g^{-1}, g))^{-1} U_g \otimes_B U_{g^{-1}} \\ &= \sum_{g \in G} \sum_{i=1}^m a_i g(b_i)(f(g^{-1}, g))^{-1} U_g \otimes_B U_{g^{-1}}. \end{aligned}$$

This implies that $\sum_{i=1}^m a_i g(b_i)(f(g^{-1}, g))^{-1} = \delta_{1,g}$, so $\sum_{i=1}^m a_i g(b_i) = \delta_{1,g}$, that is, $\{a_i, b_i \mid i = 1, 2, \dots, m\}$ is a G -Galois system for C . Therefore, C is a Galois algebra over C^G with Galois group $G|_C \cong G$.

(\Leftarrow) Since C is a Galois algebra over C^G with Galois group with $G|_C \cong G$, there exists a G -Galois system $\{a_i, b_i \in C \mid i = 1, 2, \dots, m\}$ for some integer m such that $\sum_{i=1}^m a_i g(b_i) = \delta_{1,g}$. Let $x_i = a_i$ and $y_i = \sum_{g \in G} g(b_i) U_g \otimes_B U_g^{-1}$. We claim that $\{x_i \in V_\Delta(B), y_i \in V_{\Delta \otimes_B \Delta}(\Delta) \mid i = 1, 2, \dots, m\}$ is a H -separable system for Δ over B . In fact, $x_i = a_i \in C \subset V_\Delta(B)$. Noting that $U_g^{-1} = f(g, g^{-1})^{-1} U_{g^{-1}}$, we have $U_g^{-1} b = f(g, g^{-1})^{-1} U_{g^{-1}} b = f(g, g^{-1})^{-1} g^{-1}(b) U_{g^{-1}} = g^{-1}(b) f(g, g^{-1})^{-1} U_{g^{-1}} = g^{-1}(b) U_g^{-1}$ for any $b \in B$. Hence

$$\begin{aligned} b y_i &= b \sum_{g \in G} g(b_i) U_g \otimes_B U_g^{-1} = \sum_{g \in G} g(b_i) b U_g \otimes_B U_g^{-1} \\ &= \sum_{g \in G} g(b_i) U_g g^{-1}(b) \otimes_B U_g^{-1} = \sum_{g \in G} g(b_i) U_g \otimes_B g^{-1}(b) U_g^{-1} \\ &= \sum_{g \in G} g(b_i) U_g \otimes_B U_g^{-1} b = y_i b. \end{aligned}$$

For any $h \in G$,

$$\begin{aligned}
U_h y_i &= U_h \sum_{g \in G} g(b_i) U_g \otimes_B U_g^{-1} = \sum_{g \in G} (hg)(b_i) U_h U_g \otimes_B U_g^{-1} \\
&= \sum_{g \in G} (hg)(b_i) f(h, g) U_{hg} \otimes_B U_g^{-1} = \sum_{g \in G} (hg)(b_i) U_{hg} \otimes_B f(h, g) U_g^{-1} \\
&= \sum_{g \in G} (hg)(b_i) U_{hg} \otimes_B U_{hg}^{-1} U_{hg} f(h, g) U_g^{-1} \\
&= \sum_{g \in G} (hg)(b_i) U_{hg} \otimes_B U_{hg}^{-1} U_h U_g U_g^{-1} = \sum_{g \in G} (hg)(b_i) U_{hg} \otimes_B U_{hg}^{-1} U_h \\
&= \sum_{k \in G} k(b_i) U_k \otimes_B U_k^{-1} U_h \\
&= y_i U_h.
\end{aligned}$$

Thus $y_i \in V_{\Delta \otimes_B \Delta}(\Delta)$. Moreover, $\sum_{i=1}^m x_i y_i = \sum_{i=1}^m a_i \sum_{g \in G} g(b_i) U_g \otimes_B U_g^{-1} = \sum_{g \in G} \sum_{i=1}^m a_i g(b_i) U_g \otimes_B U_g^{-1} = \sum_{g \in G} \delta_{1,g} U_g \otimes_B U_g^{-1} = 1 \otimes 1$. This implies that $\{x_i \in V_{\Delta}(B), y_i \in V_{\Delta \otimes_B \Delta}(\Delta) \mid i = 1, 2, \dots, m\}$ is a H -separable system for Δ over B . Thus, Δ is a H -separable extension of B . Moreover, B is a direct summand of Δ as a left B -module, so $V_{\Delta}(V_{\Delta}(B)) = B$ ([7], Proposition 1.2). But then, the center of Δ , $Z \subset B$; and so $Z = C^G$. Clearly, $V_{\Delta}(B)^{\bar{G}} = Z = C^G$ and $C \subset V_{\Delta}(B)$, so $V_{\Delta}(B)$ is a G -Galois algebra over C^G with the same Galois system as C . Therefore, $V_{\Delta}(B) = C$ which is commutative. The proof is completed.

The Ikehata theorem is an immediate consequence of Theorem 3.2 by the fact that any Galois algebra with a cyclic Galois group is a commutative ring ([2], Theorem 11).

COROLLARY 3.3 (The Ikehata theorem). Let ρ be an automorphism of B of order n and $B[x; \rho]$ a skew polynomial ring of degree n with $x^n = v \in U(B^\rho)$ for some integer n . Then, $B[x; \rho]$ is a H -separable extension of B if and only if C is a Galois algebra over C^ρ with Galois group $\langle \rho|_C \rangle \cong \langle \rho \rangle$.

PROOF. It is easy to check that if ρ has order n , then $x^n = v \in U(C^\rho)$. Let $B[x; \rho]$ be a H -separable extension of B . Then $V_{B[x; \rho]}(B)$ is a Galois algebra over C^ρ with cyclic

Galois group $\langle \bar{\rho} \rangle$ generated by $\bar{\rho}$ ([6], Theorem 3.2); and so $V_{B[x;\rho]}(B)$ is a commutative ring by ([2], Theorem 11). On the other hand, $B[x;\rho]$ is a crossed product $\Delta(B, \langle \rho \rangle, f)$ where $f : \langle \rho \rangle \times \langle \rho \rangle \rightarrow U(C^\rho)$ by $f(\rho^i, \rho^j) = 1$ if $i + j < n$, $f(\rho^i, \rho^j) = v$ if $i + j \geq n$, and $U_{\rho^i} = x^i$ for $i = 0, 1, 2, \dots, n - 1$. Thus the Corollary is immediate from Theorem 3.2.

Next we prove more characterizations of the ring B as given in Theorem 3.2.

THEOREM 3.4. Assume Δ is a H -separable extension of B . Then the following statements are equivalent:

- (1) $V_\Delta(B)$ is a commutative subring of Δ .
- (2) $V_\Delta(B) = C$.
- (3) $V_\Delta(C) = B$.
- (4) $J_g = \{0\}$ for each $g \neq 1$ where $J_g = \{b \in B \mid ab = bg(a) \text{ for all } a \in B\}$.
- (5) $I_g = \{0\}$ for each $g \neq 1$ where $I_g = \{b \in B \mid cb = bg(c) \text{ for all } c \in C\}$.

PROOF. We shall prove (1) \implies (2) \implies (3) \implies (4) \implies (5) \implies (1).

(1) \implies (2). This was given in the proof of the necessity of Theorem 3.2.

(2) \implies (3). Clearly, $B \subseteq V_\Delta(C)$. Conversely, for each $\sum_{g \in G} b_g U_g$ in $V_\Delta(C)$, we have $c(\sum_{g \in G} b_g U_g) = (\sum_{g \in G} b_g U_g)c$ for each c in C , so $cb_g = b_g g(c)$, that is, $b_g(c - g(c)) = 0$ for each $g \in G$ and $c \in C$. But C is a commutative G -Galois extension of C^G , so the ideal of C generated by $\{c - g(c) \mid c \in C\}$ is C when $g \neq 1$ ([1], Proposition 1.2-(5)). Hence $b_g = 0$ for each $g \neq 1$. But then $\sum_{g \in G} b_g U_g = b_1 \in B$. Thus $V_\Delta(C) \subseteq B$, and so $V_\Delta(C) = B$.

(3) \implies (4). By hypothesis, $V_\Delta(C) = B$, so $V_\Delta(B) \subset V_\Delta(C) = B$. But $V_\Delta(B) = \sum_{g \in G} J_g U_g$ by Lemma 3.1-(a), so $\sum_{g \in G} J_g U_g = V_\Delta(B) \subset B$. Thus $J_g = \{0\}$ for each $g \neq 1$.

(4) \implies (5). By Lemma 3.1-(a) again, $V_\Delta(B) = \sum_{g \in G} J_g U_g$, and by hypothesis, $J_g = \{0\}$ for each $g \neq 1$, so $V_\Delta(B) = J_1 = C$. Hence part (2) holds; and so $V_\Delta(C) = B$ by (2) \implies (3). Clearly, $V_\Delta(C) = \sum_{g \in G} I_g U_g$, so $\sum_{g \in G} I_g U_g = B$. Thus $I_g = \{0\}$ for each $g \neq 1$.

(5) \implies (1). Since $C \subset B$, $J_g \subset I_g$ for all $g \in G$. Hence $I_g = \{0\}$ implies $J_g = \{0\}$. But then $V_\Delta(B) = \sum_{g \in G} J_g U_g = J_1 = C$ which is commutative.

COROLLARY 3.5. C is a Galois algebra over C^G with Galois group $G|_C \cong G$ if and only if Δ is a H -separable extension of B and any one of the equivalent conditions in Theorem 3.4 holds.

We conclude the present paper with two examples of crossed products Δ to demonstrate our results: (1) Δ is a H -separable extension of B , but $V_\Delta(B)$ is not commutative and (2) $V_\Delta(B)$ is commutative, but Δ is not a H -separable extension of B . Hence C is not a Galois algebra over C^G with $G|_C \cong G$ in either example by Theorem 3.2.

EXAMPLE 1. Let $B = Q[i, j, k] = Q + Qi + Qj + Qk$ be the quaternion algebra over the rational field Q , $G = \{g_1 = 1, g_i, g_j, g_k \mid g_i(x) = xix^{-1}, g_j(x) = xjx^{-1}, g_k(x) = xkx^{-1}$ for all $x \in B\}$, and $\Delta = \Delta(B, G, 1)$. Then

(1) The center of Δ , $Z = Q = C$, the center of B .

(2) Δ is a separable extension of B and B is an Azumaya Q -algebra, so Δ is an Azumaya Q -algebra. Since Δ is a free left B -module, Δ is a H -separable extension of B ([4], Theorem 1).

(3) $V_\Delta(B) = Q + QiU_{g_i} + QjU_{g_j} + QkU_{g_k}$ which is not commutative, so C is not a Galois algebra over C^G with Galois group $G|_C \cong G$ by Theorem 3.2.

EXAMPLE 2. Let $B = Q[i, j, k] = Q + Qi + Qj + Qk$ be the quaternion algebra over the rational field Q , $G = \{g_1 = 1, g_i \mid g_i(x) = xix^{-1}$ for all $x \in B\}$, and $\Delta = \Delta(B, G, 1)$. Then

(1) The center of B , $C = Q = C^G$.

(2) $V_\Delta(B) = Q + QiU_{g_i}$ which is commutative.

(3) The center of Δ , $Z = Q + QiU_{g_i} \neq C^G$. On the other hand, assume that Δ is a H -separable extension of B . Since B is a direct summand of Δ as a left B -module,

$V_{\Delta}(V_{\Delta}(B)) = B$ ([7], Proposition 1.2). This implies that the center of Δ , $Z = C^G$, a contradiction. Thus Δ is not a H -separable extension of B . Therefore, C is not a G -Galois algebra over C^G with $G|_C \cong G$ by Theorem 3.2.

REFERENCES

- [1] DeMeyer F.R. and Ingraham E., Separable Algebras over Commutative Rings, Volume 181, Springer Verlag, Berlin, Heidelberg, New York, 1971.
- [2] DeMeyer F.R., Some Notes on the General Galois Theory of Rings, Osaka J. Math., 2: 117-127, 1965.
- [3] Ikehata S., On H -separable polynomials of prime degree, Math. J. Okayama Univ. 33: 21-26, 1991.
- [4] Ikehata S., Note on Azumaya algebras and H -separable extensions, Math. J. Okayama Univ. 23: 17-18, 1981.
- [5] Ikehata S. and Szeto G., On H -separable polynomials in skew polynomial rings of automorphism type, Math. J. Okayama Univ. 34: 49-55, 1992.
- [6] Ikehata S. and Szeto G., On H -skew polynomial rings and Galois extensions, Rings, Extension and Cohomology, 113-121, Lecture Notes in Pure and Appl. Math., 159, Dekker, New York, 1994.
- [7] Sugano K., Note on semisimple extensions and separable extensions, Osaka J. Math., 4: 265-270, 1967.
- [8] Szeto G. and Xue L., On the Ikehata theorem for H -separable skew polynomial Rings, Mathematical Journal of Okayama University, to appear.