

Math 310: Number Theory

Explain your solutions to the problems below clearly, completely, and concisely.

1. Let (a, b, c) be a primitive Pythagorean triple. Show that one of a or b is even, but not both.
2. Let $g(n) = \begin{cases} 0 & \text{if } n \text{ even} \\ 1 & \text{if } n \text{ odd} \end{cases}$. Show that g is multiplicative, find a closed form (i.e. formula) for g , and characterize its values.

For the next three problems, let \mathcal{M} be the set of multiplicative functions and define $\mathcal{F} : \mathcal{M} \rightarrow \mathcal{M}$ via $\mathcal{F}(f)(n) = \sum_{d|n} f(d)$.

3. Is \mathcal{F} injective? (Hint: Möbius inversion.)
4. Is \mathcal{F} surjective? (Hint: Let $f \in \mathcal{M}$. Define $F(n) = \sum_{d|n} \mu(d)f(n/d)$. What is $\mathcal{F}(F)$?)
5. Is \mathcal{F} bijective? (Hint: Ask Emma.)