Math 122: Applications of Differential Equations

1. A certain small country has \$10 billion in paper currency in circulation, and each day \$50 million comes into the country's banks. The government decides to introduce new currency by having the banks replace old bills with new ones whenever old currency comes into the banks. Let $x = x(t)$ denote the amount of new currency in circulation at time t and $x(0) = 0$.
(a) Write a differential equation that represents the flow of the new currency into circulation. Be sure to include initial values.
(b) Solve this initial value problem.
(c) How long will it take for the new bills to account for 90% of the currency in circulation?

2. Let $f(t)$ be the number of students at Bradley University who have gotten a flu shot at time t. The rate of change of $f(t)$ is proportional to the number of BU students who have not yet been vaccinated. BU has 6000 students; a week ago 1500 of them had been vaccinated, and 3000 have been vaccinated as of today.
(a) Set up a differential equation that models the number of students who have gotten a flu vaccine at time t. Include all known initial values.
(b) Solve this differential equation.
(c) How long will it take before 4500 students are vaccinated?

3. An object of mass n falling near the surface of the earth is slowed by air resistance proportional to its velocity; according to Newton's second law of motion

$$m\frac{dv}{dt} = mg - kv,$$

where $v = v(t)$ is the velocity of the object at time t, and g is the acceleration of gravity near the surface of the earth.
(a) Assuming the object falls from rest at time $t = 0$ —that is, $v(0) = 0$— find the velocity at time t before it hits the ground.
(b) Show $v(t)$ approaches a limiting velocity, known as the terminal velocity, as $t \to \infty$.

4. Fred has a 400 gallon tank initially containing 100 gallons of fresh water. He starts pumping in 3 gallons of water per minute of a solution containing 1 pound of salt per gallon. He also pumps the liquid out of the tank at a rate of 1 gallon per minute.
(a) Write a differential equation describing the amount of salt in the tank at time t. Include any initial conditions.
(b) What is the amount of salt in the tank when the tank reaches its capacity?
(c) If the tank had infinite capacity, would there be a limiting concentration of salt in the tank? If so, what is that concentration?